

# Analytic Hierarchy Process for Survey Data in R

## Vignettes for the `ahpsurvey` package (ver 0.4.0)

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## Introduction

The Analytic Hierarchy Process (AHP), introduced by Saaty (1987), is a versatile multi-criteria decision-making tool that allows individuals to rationally weigh attributes and evaluate alternatives presented to them. While most applications of the AHP are focused on implementation at the individual or small-scale, the AHP was increasingly adopted in survey designs, which involve a large number of decision-makers and a great deal of heterogeneity in responses. The tools currently available in R for the analysis of AHP data, such as the packages `ahp` by Glur (2018) and `Prize` by Dargahi (2016), are excellent tools for performing the AHP at a small scale and offers are excellent in terms of interactivity, user-friendliness, and for comparing alternatives.

However, researchers looking to adopt the AHP in the analysis of survey data often have to manually reformat their data, sometimes even involving dragging and copying across Excel spreadsheets, which is painstaking and prone to human error. Hitherto, there are no good ways of computing and visualising the heterogeneity amongst AHP decision-makers, which is common in survey data. Inconsistent choices are also prevalent in AHP conducted in the survey format, where it is impractical for enumerators to identify and correct for inconsistent responses on the spot when the surveys are delivered in paper format. Even if an electronic version that allows immediate feedback of consistency ratio is used, respondents asked to repeatedly change their answers are likely to be mentally fatigued. Censoring observations with inconsistency is likely to result in a greatly decreased statistical power of the sample, or may lead to unrepresentative samples and nonresponse bias.

The `ahpsurvey` package provides a workflow for researchers to quantify and visualise inconsistent pairwise comparisons that aids researchers in improving the AHP design and adopting appropriate analytical methods for the AHP.

Install the package directly from CRAN:

```
install.packages("ahpsurvey")
```

And load the `ahpsurvey` library.

```
library(ahpsurvey)
```

**If you are familiar with the AHP and want a rapid way of processing your data, skip to the part about the canned routine towards the end.**

## The Saaty AHP and the toy dataset

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A gentle introduction of the AHP survey methodology:

Rating	Definition
1	Two characteristics are equally important
2	Between 1 and 3
3	The preferred characteristics are slightly more important
4	Between 3 and 5
5	The preferred characteristics are moderately more important
6	Between 5 and 7
7	The preferred characteristics are strongly more important
8	Between 7 and 9
9	The preferred characteristics are absolutely more important

A Saaty scale is composed of 9 items on each end (17 options per pairwise comparison) where decision-makers are asked to indicate how much attribute/ characteristic A is more preferred to B (or vice versa),

and how much it is preferred in a 9-point scale. Respondents are asked to make pairwise comparisons for a range of attributes, and indicate their priorities for each of them.

Afterwards, we load the data needed, `city200`, which consists of randomly generated data of 200 individuals based on the weights provided in Saaty (2004). The methodology of data generation is explained at the end of this vignette.

```
atts <- c("cult", "fam", "house", "jobs", "trans")
data(city200)
head(city200)
```

##	cult_fam	cult_house	cult_jobs	cult_trans	fam_house	fam_jobs	fam_trans
## 1	2	-2	2	-6	-4	-4	-8
## 2	2	-4	1	-4	-4	-2	-8
## 3	4	-2	1	-3	-7	-3	-5
## 4	8	-4	3	-4	-8	1	-7
## 5	3	-3	5	-6	-8	1	-4
## 6	6	-4	2	-4	-7	-2	-4

##	house_jobs	house_trans	jobs_trans
## 1	4	-3	-8
## 2	4	-3	-7
## 3	4	-3	-6
## 4	4	-3	-9
## 5	4	-3	-6
## 6	4	-3	-6

The simulated dataset consists of ten pairwise comparisons of five attributes, which are “culture”, “family”, “housing”, “jobs” and “transportation.” An individual compares the attributes in a pairwise fashion; if culture is more important than house by 2 units on the Saaty scale, the dataset will code it as -2. This is important to bear in mind as we move on.

# Individual and Aggregated priorities

## Creating pairwise comparison matrices

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Based on the Saaty scale, a pairwise comparison matrix of  $N$  attributes for the  $k^{th}$  individual is obtained:

$$\mathbf{S}_k = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,N} \\ \vdots & \vdots & a_{i,j} & \vdots \\ a_{N,1} & a_{N,2} & \cdots & a_{N,N} \end{pmatrix}$$

Where  $a_{i,j}$  represents the pairwise comparison between the attribute  $i$  and  $j$ . If  $i$  is more important than  $j$  for 6 units,  $a_{i,j} = 6$  and  $a_{j,i} = \frac{1}{6}$ , i.e. the reciprocal. Data must be reformatted into this pairwise comparison matrix format to proceed.

The reformatting of the survey data (with one row per individual) into such a matrix necessary for further analysis is cumbersome for researchers. Furthermore, as researchers conducting the AHP as an integrated part of a survey, we typically receive data in the above format: the pairwise comparisons are coded in positive and negative numbers as opposed to reciprocals. In the pairwise comparison of `cult_fam`:

In the case where the decision-maker chose 6, the sensible codebook maker would code it as -6, which denotes that *Culture* is more important than *Family* in 6 units for that decision-maker. For `ahp.mat` to work, the value in `A_B` variable have to be the importance A has over B in positive values. In this case, the values should be converted from negative to positive, and the negative values would be converted to its reciprocal in the pairwise matrix. When data is coded in the above way, set `negconvert = TRUE`. If the data is already coded in the reciprocal (as opposed to negatives), set `reciprocal = FALSE`.

Some caveats prior to entering the data into the `ahp.mat` function. `ahp.mat` does not recognise the names of the original dataframe, and figures out which attribute corresponds to which entirely based on the order of the columns. For example, when the attributes are A, B, C and D, the dataframe should be ordered in `A_B`, `A_C`, `A_D`, `B_C`, `B_D`, `C_D`, and the attributes listed as `c(A,B,C,D)`, in that order.

`ahp.mat` takes four arguments:

- `df`: the dataframe
- `atts`: a list of attributes in the correct order
- `negconvert`: whether to convert all positive values to negative (logical, defaults to `FALSE`)
- `reciprocal`: whether to convert negative values (after `negconvert`) to its reciprocals (defaults to `TRUE`).

```
city200 %>%
  ahp.mat(atts = atts, negconvert = TRUE) %>%
  head(3)
```

```
## [[1]]
##      cult   fam house jobs trans
```

```

## cult  1.000 0.500 2.000 0.500    6
## fam   2.000 1.000 4.000 4.000    8
## house 0.500 0.250 1.000 0.250    3
## jobs  2.000 0.250 4.000 1.000    8
## trans 0.167 0.125 0.333 0.125    1
##
## [[2]]
##      cult   fam house jobs trans
## cult  1.00 0.500 4.000 1.000    4
## fam   2.00 1.000 4.000 2.000    8
## house 0.25 0.250 1.000 0.250    3
## jobs  1.00 0.500 4.000 1.000    7
## trans 0.25 0.125 0.333 0.143    1
##
## [[3]]
##      cult   fam house jobs trans
## cult  1.000 0.250 2.000 1.000    3
## fam   4.000 1.000 7.000 3.000    5
## house 0.500 0.143 1.000 0.250    3
## jobs  1.000 0.333 4.000 1.000    6
## trans 0.333 0.200 0.333 0.167    1

```

The `ahp.mat` function creates a list of pairwise comparison matrices for all decision-makers. As seen above, the pairwise matrices resembles the original Saaty criteria weights, which is a good sanity check.

## Individual preference weights

---

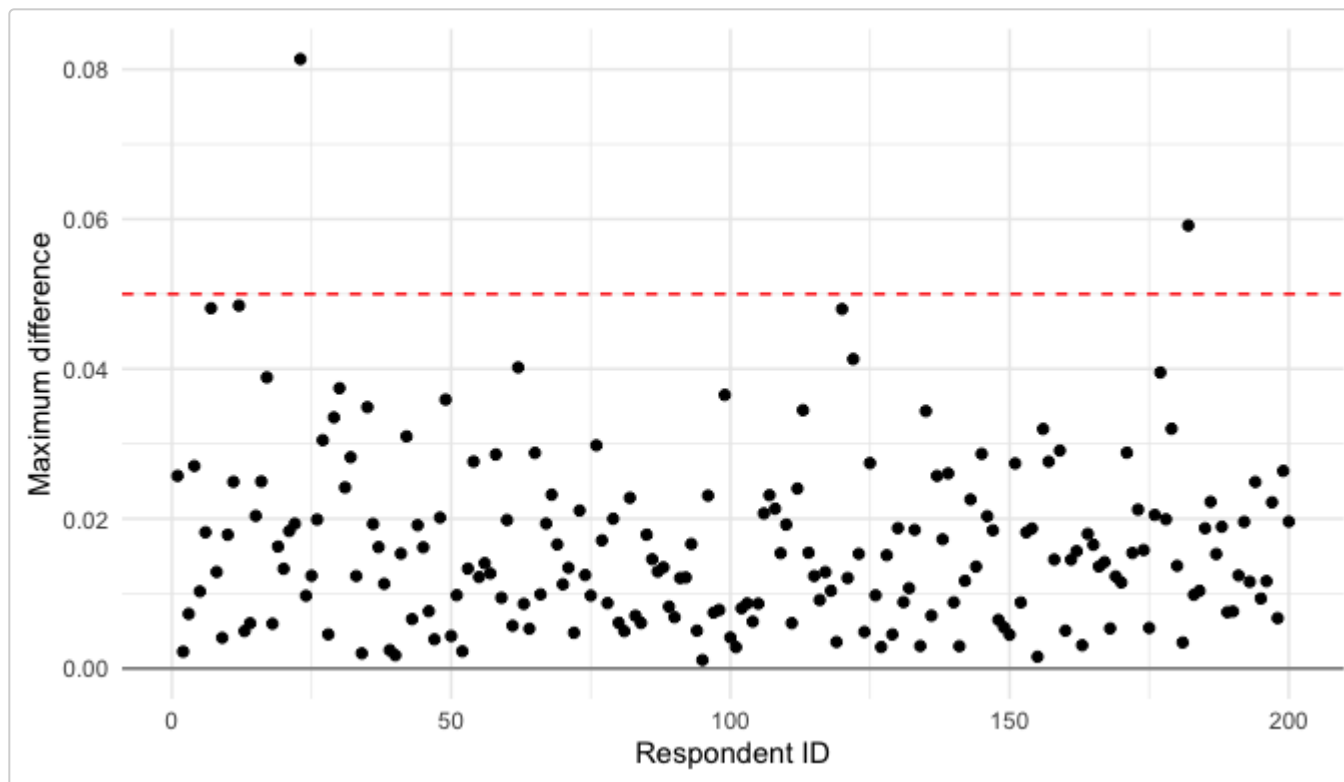
The `ahp.indpref` function computes the individual priorities of the decision-makers, and returns a `data.frame` containing the preference weights of the decision-makers. The three arguments are as follows:

- `ahpmat`: The list of matrices created by `ahp.mat`.
- `atts`: a list of attributes in the correct order.
- `method`: It normalises the matrices so that all of the columns add up to 1, and then computes the averages of the row as the preference weights of each attribute. Four modes of finding the averages are available:
  - `arithmetic`: the arithmetic mean
  - `geometric`: the geometric mean
  - `rootmean`: the square root of the sum of the squared value

- eigen: the individual preference weights are computed using the Dominant Eigenvalues method described in Saaty (2003)

Here I demonstrate the difference of using arithmetic aggregation and dominant eigenvalue methods. In my own testing with real datasets, a much higher proportion of respondents have at least one attribute with a difference larger than 0.05 due to presence of inconsistent and heterogeneous responses.

```
cityahp <- city200 %>%
  ahp.mat(atts, negconvert = T)
eigentrue <- ahp.indpref(cityahp, atts, method = "eigen")
geom <- ahp.indpref(cityahp, atts, method = "arithmetic")
error <- data.frame(id = 1:length(cityahp), maxdiff = apply(abs(eigentrue - geom), 1, max))
error %>%
  ggplot(aes(x = id, y = maxdiff)) +
  geom_point() +
  geom_hline(yintercept = 0.05, linetype = "dashed", color = "red") +
  geom_hline(yintercept = 0, color = "gray50") +
  scale_x_continuous("Respondent ID") +
  scale_y_continuous("Maximum difference") +
  theme_minimal()
```



Maximum difference of between eigenvalue and mean aggregation

## Aggregated preference weights

The `ahp.aggpref` function computes the aggregated priorities of all decision-makers using the specified methods. The following arguments are given:

- `method`: Same as `ahp.indpref`. It normalises the matrices so that all of the columns add up to 1, and then computes the averages of the row as the preference weights of each attribute. Four modes of finding the averages are available:
  - `arithmetic`: the arithmetic mean
  - `geometric`: the geometric mean
  - `rootmean`: the square root of the sum of the squared value
  - `eigen`: the individual preference weights are computed using the Dominant Eigenvalues method described in Saaty (2003)
- `aggmethod`: how to aggregate the individual priorities.
  - `arithmetic`, `geometric` and `rootmean` (same principle as `method`)
  - `tmean` trimmed mea

- `tgmean` trimmed geometric mean
- `sd` returns the standard deviation from the arithmetic mean.

When `tmean` or `tgmean` is specified, `ahpsurvey` needs an additional argument `qt`, which specifies the quantile which the top **and** bottom preference weights are trimmed. `qt = 0.25` specifies that the aggregation is the arithmetic mean of the values from the 25 to 75 percentile. This visualisation offers researchers a good way to determine the amount of preference weights to be trimmed. By default, `qt = 0`, hence the result you would get by using `tmean` and `tgmean` and not specifying `qt` is the same as `arithmetic` and `geometric` respectively.

```
amean <- ahp.aggpref(cityahp, atts, method = "arithmetic")
amean

##   cult   fam  house  jobs  trans
## 0.1620 0.4367 0.0761 0.2827 0.0424
```

Two steps were simultaneously conducted in the above command:

1. Compute the individual priorities of each decision-maker (using `method`)
2. Aggregate the priorities (using `aggmethod`)

By default, the two steps rely on the same aggregation method as specified in `method` (unless when `method = "eigen"`, where `aggmethod` defaults to `arithmetic`). However, it is possible to specify different aggregation methods for the individual and group level. For instance, one can specify that in the individual level, the arithmetic mean is used to compute the individual priorities; the priorities are aggregated using a trimmed mean by trimming observations higher and lower quantile.

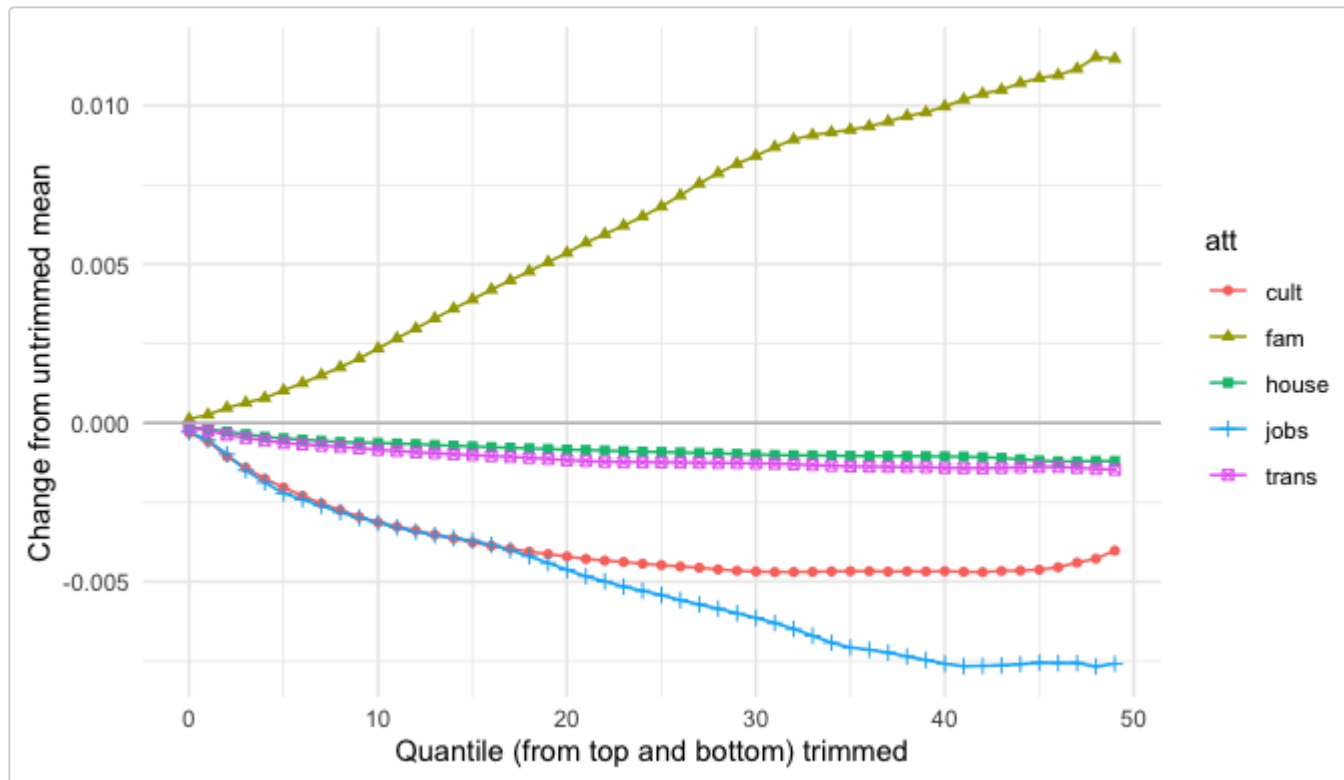
```
qtresults <- matrix(nrow = 50, ncol = 5, data = NA)
for (q in 1:50){
  qtresults[q,] <- ahp.aggpref(cityahp, atts, method = "arithmetic",
                              aggmethod = "tmean", qt = (q-1)/100)
}
colnames(qtresults) <- atts
qtresults %>%
  as.data.frame() %>%
  mutate(trimperc = 1:nrow(qtresults)-1) %>%
  mutate(cult = cult - amean[1],
         fam = fam - amean[2],
         house = house - amean[3],
```



```

jobs = jobs - amean[4],
trans = trans - amean[5]) %>%
gather(cult, fam, house, jobs, trans, key = "att", value = "weight") %>%
ggplot(aes(x = trimperc, y = weight, group = att, shape = att, color = att, fill = att)) +
geom_line() +
geom_point() +
scale_x_continuous("Quantile (from top and bottom) trimmed") +
scale_y_continuous("Change from untrimmed mean") +
geom_hline(yintercept = 0, color = "gray") +
theme_minimal()

```



Changes of aggregated weights based on quantile of data trimmed

It is also possible to quantify the heterogeneity amongst decision-makers' priorities, information possibly lost by group aggregation. This is specified using `aggmethod = "sd"`:

```

mean <- city200 %>%
  ahp.mat(att = atts, negconvert = TRUE) %>%

```

```

ahp.aggpref(atts, method = "arithmetic")

sd <- city200 %>%
  ahp.mat(atts = atts, negconvert = TRUE) %>%
  ahp.aggpref(atts, method = "arithmetic", aggmethod = "sd")

t(data.frame(mean, sd))%>% kable()

```

	cult	fam	house	jobs	trans
mean	0.162	0.437	0.076	0.283	0.042
sd	0.033	0.054	0.009	0.048	0.007

## Aggregated individual judgements

Similarly, `ahp.aggjudge` aggregates the individual judgements of all decision-makers to generate a row-standardised pairwise comparison matrix of all decision-makers. This allows one to compare priorities directly based on the aggregated pairwise judgements of all decision-makers. It takes the argument `aggmethod` with the exact same options as `ahp.aggpref`.

```

city200 %>%
  ahp.mat(atts = atts, negconvert = TRUE) %>%
  ahp.aggjudge(atts, aggmethod = "geometric")

##      cult   fam house jobs trans
## cult  1.000 0.220 3.093 0.488  4.64
## fam   4.541 1.000 6.461 1.704  6.15
## house 0.323 0.155 1.000 0.249  2.93
## jobs  2.048 0.587 4.019 1.000  7.04
## trans 0.216 0.163 0.342 0.142  1.00

```

# Measuring and visualising consistency

## Measuring consistency

The consistency indices and consistency ratio of a given choice is defined by the following equation:

$$CR = \left( \frac{\lambda_{max} - n}{n - 1} \right) \left( \frac{1}{RI} \right)$$

Where  $\lambda_{max}$  is the maximum eigenvalue of the pairwise comparison vector and  $n$  is the number of attributes. The  $RI$  when five attributes are present is 1.11. See the documentation for `ahp.ri` to generate your own  $RI$  based on a specific number of dimensions and random seed.

The  $RI$  below was generated from `ahp.ri` with 500000 simulations (which takes some time), as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.525	0.884	1.11	1.25	1.34	1.41	1.45	1.49	1.51	1.54	1.55	1.57	1.58

Saaty showed that when the  $CR$  is higher than 0.1, the choice is deemed to be inconsistent. The `ahpsurvey` package allows researchers to quantify the inconsistency among the decision-makers and make decisions in their analysis, either to drop inconsistent observations or look for ways to adjust for inconsistency. As a proof of concept, I use the original weights to compute the consistency ratio, and it returned the value which Saaty got, 0.05.

```
weight <- c(5, -3, 2, -5,
           -7, -1, -7,
           4, -3,
           -7)
sample_mat <- ahp.mat(t(weight), atts, negconvert = TRUE)

(cr_std <- ahp.cr(sample_mat, atts))

## [1] 0.0508
```

The `ahp.cr` function returns a vector of  $CR$  that can be merged to other dataframes as a measure of the individuals' consistency.

```
cr <- city200 %>%
  ahp.mat(atts, negconvert = T) %>%
  ahp.cr(atts)
table(cr <= 0.1)
```

```
##
## FALSE TRUE
## 70 130
```

You may also specify your own random index generated with `ahp.ri` to be used with `ahp.cr`, as follows:

```
## Generate a random index with 1000 simulations, 5 dimensions and seed 30000 for reproducibility
  (seed = 42 by default).
(RI <- ahp.ri(nsims = 1000, dim = 5, seed = 30000))
```

```
## [1] 1.12
```

```
## Use this RI to calculate the consistency ratio instead of the default one.
ahp.cr(sample_mat, atts, RI)
```

```
## [1] 0.0505
```

The processing time of `ahp.ri` increases exponentially as `nsims` increase, and unfortunately I haven't written it to optimise for speed. Generally I wouldn't go beyond 6 digits of `nsims` unless I have too much time lying around.

## Visualising individual priorities and consistency ratios

The `ahp.indpref` function provides a detailed account of each individuals' priorities and its corresponding weighting. An overlay of the violin density, boxplots and jitter plots is useful in visualising the heterogeneity in weights each respondent gives.

```
thres <- 0.1
dict <- c("cult" = "Culture",
         "fam" = "Family",
         "house" = "Housing",
         "jobs" = "Jobs",
         "trans" = "Transportation")

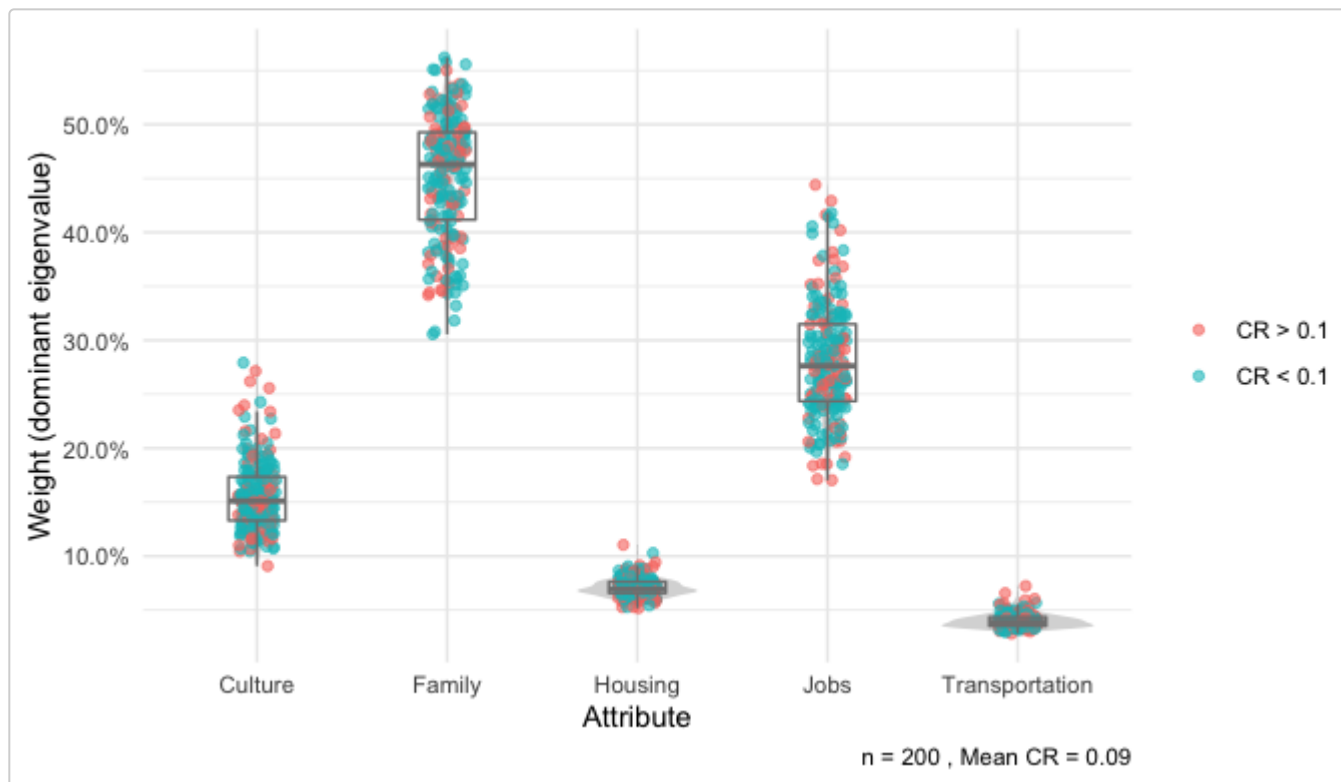
cr.df <- city200 %>%
  ahp.mat(atts, negconvert = TRUE) %>%
```

```

ahp.cr(atts) %>%
data.frame() %>%
mutate(rowid = 1:length(cr), cr.dum = as.factor(ifelse(cr <= thres, 1, 0))) %>%
select(cr.dum, rowid)

city200 %>%
  ahp.mat(atts = atts, negconvert = TRUE) %>%
  ahp.indpref(atts, method = "eigen") %>%
  mutate(rowid = 1:nrow(eigentrue)) %>%
  left_join(cr.df, by = 'rowid') %>%
  gather(cult, fam, house, jobs, trans, key = "var", value = "pref") %>%
  ggplot(aes(x = var, y = pref)) +
  geom_violin(alpha = 0.6, width = 0.8, color = "transparent", fill = "gray") +
  geom_jitter(alpha = 0.6, height = 0, width = 0.1, aes(color = cr.dum)) +
  geom_boxplot(alpha = 0, width = 0.3, color = "#808080") +
  scale_x_discrete("Attribute", label = dict) +
  scale_y_continuous("Weight (dominant eigenvalue)",
                    labels = scales::percent,
                    breaks = c(seq(0,0.7,0.1))) +
  guides(color=guide_legend(title=NULL))+
  scale_color_discrete(breaks = c(0,1),
                    labels = c(paste("CR >", thres),
                               paste("CR <", thres))) +
  labs(NULL, caption = paste("n =", nrow(city200), ",", "Mean CR =",
                              round(mean(cr),3)))+
  theme_minimal()

```



# Dealing with inconsistent and missing data

## Identifying inconsistent pairwise comparisons

Not only are survey designers interested in the level of inconsistency present in their surveys, they are also interested in the source of inconsistency. Are respondents making inconsistent choices because some attributes are ill-defined, or that a pairwise comparison between those attributes simply do not make sense? `ahpsurvey` provides easy tools for researchers to identify the pairwise comparisons which respondents make inconsistent choices, which could contribute to better survey designs.

The `ahp.pwerror` compares the pairwise matrix of each individual with a Saaty Matrix (that has the property of  $CR = 0$ ) generated using the obtained preference weights. It is always better to understand this with an example.

The Saaty matrix is defined as the following:

$$\mathbf{S} = \begin{pmatrix} \frac{p_1}{p_1} & \frac{p_1}{p_2} & \dots & \frac{p_1}{p_N} \\ \frac{p_2}{p_1} & \frac{p_2}{p_2} & \dots & \frac{p_2}{p_N} \\ \vdots & \vdots & \frac{p_i}{p_j} & \vdots \\ \frac{p_N}{p_1} & \frac{p_N}{p_2} & \dots & \frac{p_N}{p_N} \end{pmatrix}$$

Where  $p_i$  and  $p_j$  are the final weights of the  $i^{\text{th}}$  and  $j^{\text{th}}$  attribute respectively, and  $N$  is the number of attributes.

I am no math major, and I find linear algebra intimidating. Here, I will demonstrate with an example from Saaty's original matrix how we arrive the consistency error matrix from the original pairwise matrix.

Consider this matrix of the original pairwise comparison and the resultant preference weights below.

```
## [[1]]
##      cult   fam house jobs trans
## cult  1.000 0.200 3.000 0.500    5
## fam   5.000 1.000 7.000 1.000    7
## house 0.333 0.143 1.000 0.250    3
## jobs  2.000 1.000 4.000 1.000    7
## trans 0.200 0.143 0.333 0.143    1
```

The goal is to compare the above matrix with a perfectly consistent Saaty matrix generated from the preference weights calculated using the dominant eigenvalue method.

```
preference <- t(ahp.indpref(sample_mat, atts, method = "eigen"))
preference

##      [,1]
## cult 0.1522
## fam  0.4335
## house 0.0716
## jobs 0.3050
## trans 0.0378
```

The matrix is generated by multiplying its transposed reciprocal version of itself. This is no rocket science – for example, the cult fam comparison is calculated by dividing weight of cult by the weight of fam, 0.152 /

0.433 = 0.351.

```
S <- preference %*% t((preference)^-1)
S

##          cult    fam house  jobs trans
## cult  1.000 0.3511 2.127 0.499 4.02
## fam   2.849 1.0000 6.058 1.421 11.46
## house 0.470 0.1651 1.000 0.235 1.89
## jobs  2.004 0.7037 4.262 1.000 8.07
## trans 0.249 0.0872 0.528 0.124 1.00
```

The transposed Saaty matrix is multiplied element-by-element with the original pairwise comparison matrix (or taken its reciprocals if the product is smaller than 1) to generate a measure of how well the pairwise matrix resembles the Saaty matrix. If the matrix perfectly resembles the transposed Saaty matrix, the consistency error matrix (shown below) should very close to 1. This matrix is expressed as the following:

$$\epsilon_{ij} = a_{ij} \frac{p_j}{p_i}$$

Where  $a_{ij}$  is the value in the pairwise comparison matrix. The values can be obtained with a simple matrix multiplication of the transpose of **S**.

```
sample_mat[[1]] * t(S)

##          cult    fam house  jobs trans
## cult  1.000 0.570 1.411 1.002 1.243
## fam   1.755 1.000 1.156 0.704 0.611
## house 0.709 0.865 1.000 1.066 1.585
## jobs  0.998 1.421 0.938 1.000 0.868
## trans 0.805 1.637 0.631 1.152 1.000
```

The process is automated in `ahp.error`. `ahp.error` also loops through all pairwise comparison matrices generated by `ahp.mat`, and returns a list of error consistency matrices. The consistency matrices quantifies the inconsistency underlying each pairwise comparison of each decision-maker. I can also use `reciprocal = TRUE` to put all the errors that are above 1 into the upper triangular matrix. If `reciprocal = FALSE`, the below output will be essentially the same as the matrix above.



```

error <- ahp.error(sample_mat, atts, reciprocal = TRUE)
error

## [[1]]
##      cult  fam house jobs trans
## cult    1 1.76  1.41 1.00  1.24
## fam     NA 1.00  1.16 1.42  1.64
## house   NA  NA   1.00 1.07  1.59
## jobs    NA  NA    NA 1.00  1.15
## trans   NA  NA    NA  NA   1.00

```

Here I demonstrate how to perform `ahp.error` in our 200 simulated decision-makers and compute the mean consistency error for each pairwise comparison. By using `reciprocal = TRUE`, I put all the errors that are above 1 into the upper triangular matrix so that we can summarise (by taking geometric mean) quickly the average error of each pairwise comparison (larger means more error).

```

gm_mean <- function(x, na.rm=TRUE){
  exp(sum(log(x[x > 0])), na.rm=na.rm) / length(x)
}

mat <- cityahp %>%
  ahp.error(atts, reciprocal = TRUE) %>%
  unlist() %>%
  as.numeric() %>%
  array(dim=c(length(atts), length(atts), length(cityahp))) %>%
  apply(c(1,2), gm_mean)

colnames(mat) <- rownames(mat) <- atts

mat

##      cult  fam house jobs trans
## cult    1 1.89  1.55 1.92  1.29
## fam     1 1.00  1.33 1.59  2.19
## house   1 1.00  1.00 1.37  1.24
## jobs    1 1.00  1.00 1.00  1.13
## trans   1 1.00  1.00 1.00  1.00

```

The above matrix is a quick way for revealing inconsistencies within the data, but it is not the best way as it can be biased. If one or more decision-maker makes an incredibly inconsistent pairwise comparison, the consistency error for that pairwise comparison will be very high, which biases the mean error consistency of that pairwise comparison upwards even if many other decision-makers are making perfectly consistent choices.

## Finding inconsistent pairwise comparisons by maximum

A better way, as I reckon, would be to extract the pairwise comparison with the maximum inconsistency error, and returning a list of the most inconsistent pairwise comparisons for each decision-maker. This process is automated in the `ahp.pwerror` function, which returns a dataframe of the top three most inconsistent pairwise comparison made by each decision-maker.

```
city200 %>%
  ahp.mat(atts) %>%
  ahp.pwerror(atts) %>%
  head()

##           top1           top2           top3
## 1  fam_jobs house_jobs cult_jobs
## 2 cult_house house_jobs fam_trans
## 3  fam_trans  cult_fam cult_trans
## 4  cult_fam  cult_jobs cult_house
## 5 cult_jobs  fam_trans  fam_jobs
## 6 fam_trans  cult_fam cult_house
```

A better way to visualise the pairwise comparisons is a bar chart:

```
cityahp %>%
  ahp.pwerror(atts) %>%
  gather(top1, top2, top3, key = "max", value = "pair") %>%
  table() %>%
  as.data.frame() %>%
  ggplot(aes(x = pair, y = Freq, fill = max)) +
  geom_bar(stat = 'identity') +
  scale_y_continuous("Frequency", breaks = c(seq(0,180,20))) +
  scale_fill_discrete(breaks = c("top1", "top2", "top3"), labels = c("1", "2", "3")) +
  scale_x_discrete("Pair") +
```

```

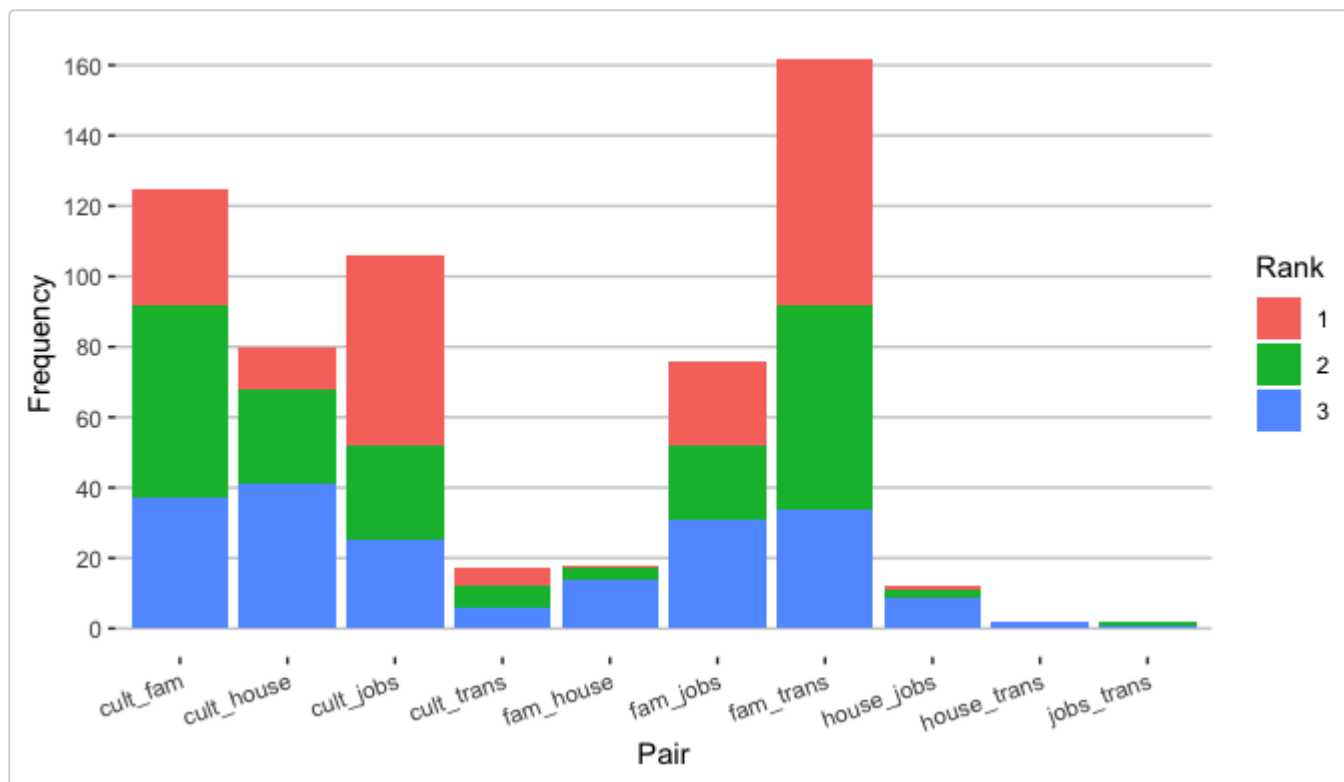
guides(fill = guide_legend(title="Rank")) +
theme(axis.text.x = element_text(angle = 20, hjust = 1),
      panel.background = element_rect(fill = NA),
      panel.grid.major.y = element_line(colour = "grey80"),
      panel.grid.major.x = element_blank(),
      panel.ontop = FALSE)

```

```

## Warning: attributes are not identical across measure variables;
## they will be dropped

```



Pairwise comparison and its frequency as the most, second-most, and third most inconsistent pairwise comparison

The results are favorable – the frequency which a pairwise comparison is the most inconsistent for that decision-maker is reflective of the degree of randomness I have used to generate the dataset. The cult\_fam, cult\_jobs and fam\_trans are assigned the highest standard deviations for the normal random draw, which partly contributes to its high frequency of being in the most inconsistent pairwise comparison in the chart.

## Transforming inconsistent matrices

Inconsistent pairwise matrices are problematic for AHP survey analysts. Harker (1987) described a method to replace inconsistent values: using the error matrix we have derived above, we can suggest a value that would reduce the inconsistency. Consider the below pairwise matrix found in Saaty's explication of Harker's method:

```
family <- c(1, 1/5, 1/3, 1/7, 1/6, 1/6, 3, 4,
           5, 1, 3, 1/5, 1/3, 1/3, 5, 7,
           3, 1/3, 1, 1/6, 1/3, 1/4, 1/6, 5,
           7, 5, 6, 1, 3, 4, 7, 8,
           6, 3, 3, 1/3, 1, 2, 5, 6,
           6, 3, 4, 1/4, 1/2, 1, 5, 6,
           1/3, 1/5, 6, 1/7, 1/5, 1/5, 1, 2,
           1/4, 1/7, 1/5, 1/8, 1/6, 1/6, 1/2, 1)

fam.mat <- list(matrix(family, nrow = 8 , ncol = 8))

atts <- c("size", "trans", "nbrhd", "age", "yard", "modern", "cond", "finance")

rownames(fam.mat[[1]]) <- colnames(fam.mat[[1]]) <- atts

fam.mat[[1]] %>% kable()
```

	size	trans	nbrhd	age	yard	modern	cond	finance
size	1.000	5.000	3.000	7	6.000	6.00	0.333	0.250
trans	0.200	1.000	0.333	5	3.000	3.00	0.200	0.143
nbrhd	0.333	3.000	1.000	6	3.000	4.00	6.000	0.200
age	0.143	0.200	0.167	1	0.333	0.25	0.143	0.125
yard	0.167	0.333	0.333	3	1.000	0.50	0.200	0.167
modern	0.167	0.333	0.250	4	2.000	1.00	0.200	0.167
cond	3.000	5.000	0.167	7	5.000	5.00	1.000	0.500
finance	4.000	7.000	5.000	8	6.000	6.00	2.000	1.000

```
ahp.cr(fam.mat, atts)
```

```
## [1] 0.17
```

The consistency ratio of the pairwise matrix is unsatisfactory. The procedure involved in Harker's method is as follows:

1. Find the pairwise comparison with the maximum error (the  $i^{th}$  and  $j^{th}$  element)
2. Duplicate the matrix and replace the pairwise comparison in the new matrix with the maximum error with 0, and its two corresponding diagonal entries with 2
3. Compute new weights  $w_i$  and  $w_j$  (as in `ahp.indpref` with `method = "eigen"`)
4. Replace the pairwise comparison with  $\frac{w_i}{w_j}$  and  $\frac{w_j}{w_i}$

For an in-depth explication see Saaty (2003). Here I replicate the results in Saaty (2003) with the `ahp.harker` function.

```
edited <- ahp.harker(fam.mat, atts, iterations = 10, stopcr = 0.1)
```

```
## [1] "Ind 1 Iterations: 1"
```

```
edited[[1]]%>% kable()
```

	size	trans	nbrhd	age	yard	modern	cond	finance
size	1.000	5.000	3.000	7	6.000	6.00	0.333	0.250
trans	0.200	1.000	0.333	5	3.000	3.00	0.200	0.143
nbrhd	0.333	3.000	1.000	6	3.000	4.00	0.459	0.200
age	0.143	0.200	0.167	1	0.333	0.25	0.143	0.125
yard	0.167	0.333	0.333	3	1.000	0.50	0.200	0.167
modern	0.167	0.333	0.250	4	2.000	1.00	0.200	0.167
cond	3.000	5.000	2.180	7	5.000	5.00	1.000	0.500
finance	4.000	7.000	5.000	8	6.000	6.00	2.000	1.000

```
ahp.cr(edited, atts)
```

```
## [1] 0.0825
```

As seen here, element [3, 7] is the most inconsistent pairwise comparison, thus it was replaced with a more consistent value 0.459.

`ahp.harker` takes five optional arguments:

- `round` is logical and tells `ahp.harker` whether to convert the newly replaced values to integers and its reciprocals, and can be set to `TRUE` if desired.
- `iterations` denotes how many pairwise comparisons should be changed. For example, if `iterations = 3`, `ahp.harker` changes the first, second, and third most inconsistent pairwise comparisons using that method. Researchers should think carefully how many pairwise comparisons should be replaced, as every time a pairwise comparison is replaced, some information is inevitably lost. Note that the maximum number of iterations is capped at  $iterations \leq \frac{1}{2}n(n - 1)$  with  $n$  being the number of attributes.
- `stopcr`: The stopping Consistency Ratio. It complements `iterations` by giving `iterations` a criteria to stop when a matrix is sufficiently consistent. `ahp.harker` will continue looping and replacing more elements of the pairwise comparison matrices until the consistency ratio of the new matrix is lower than `stopcr`, or the maximum number of iterations is reached, and will stop and move onto the next individual. When `stopcr` is set, the number of replaced elements will differ amongst each decision-maker. Thus, it is advised that the analyst set `printiter = TRUE` to see how many iterations has the pairwise matrix of that individual has been modified by the algorithm.
- `limit`: In many cases, the algorithm will intend to replace a value with a number higher than 9 or lower than 1/9. `limit` caps the maximum and minimum value of the replacement to 9 and 1/9 respectively.
- `printiter` is a logical argument of whether the number of iterations taken for each pairwise matrix is reported or not. Generally it is not needed if `stopcr` is not specified. When `stopcr` is specified, this is a good way of identifying how many pairwise comparisons are actually replaced by the algorithm for each decision maker. The printout above shows "Ind 1 Iterations: 1", which shows that although I specified `iterations = 10`, individual 1 (Ind 1) was only iterated one time before it reached the target consistency ratio, 0.1. Only one element was replaced.

I will demonstrate how `ahp.harker` improved the consistency of the decision-makers in our fictitious sample.

```

crmat <- matrix(NA, nrow = 200, ncol = 11)
colnames(crmat) <- 0:10

atts <- c("cult", "fam", "house", "jobs", "trans")

crmat[,1] <- city200 %>%
  ahp.mat(atts, negconvert = TRUE) %>%
  ahp.cr(atts)

for (it in 1:10){
  crmat[,it+1] <- city200 %>%
    ahp.mat(atts, negconvert = TRUE) %>%
    ahp.harker(atts, iterations = it, stopcr = 0.1,
              limit = T, round = T, printiter = F) %>%
    ahp.cr(atts)
}

data.frame(table(crmat[,1] <= 0.1),
           table(crmat[,3] <= 0.1),
           table(crmat[,5] <= 0.1)) %>%
  select(Var1, Freq, Freq.1, Freq.2) %>%
  rename("Consistent?" = "Var1", "No Iteration" = "Freq",
        "2 Iterations" = "Freq.1", "4 Iterations" = "Freq.2")

## Consistent? No Iteration 2 Iterations 4 Iterations
## 1 FALSE 70 13 2
## 2 TRUE 130 187 198

```

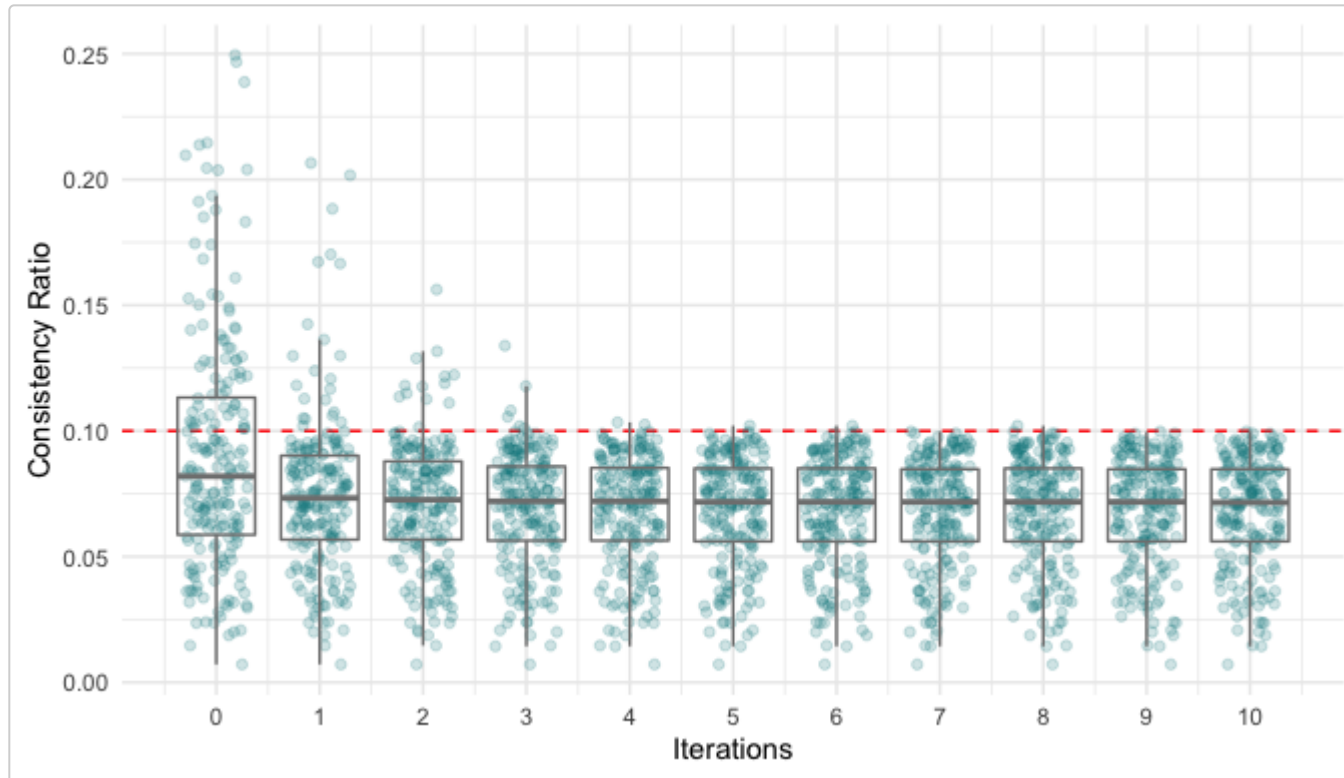
While using Harker's method cannot completely lower the CR of all decision-makers to desired levels, it allows researchers to keep a lot more observations; whereas we would have to truncate 70 samples, now we only have to censor 22 samples with 1 iteration.

```

crmat %>%
  as.data.frame() %>%
  gather(key = "iter", value = "cr", `0`, 1,2,3,4,5,6,7,8,9,10,11) %>%
  mutate(iter = as.integer(iter)) %>%
  ggplot(aes(x = iter, y = cr, group = iter)) +
  geom_hline(yintercept = 0.1, color = "red", linetype = "dashed")+

```

```
geom_jitter(alpha = 0.2, width = 0.3, height = 0, color = "turquoise4") +
geom_boxplot(fill = "transparent", color = "#808080", outlier.shape = NA) +
scale_x_continuous("Iterations", breaks = 0:10) +
scale_y_continuous("Consistency Ratio") +
theme_minimal()
```



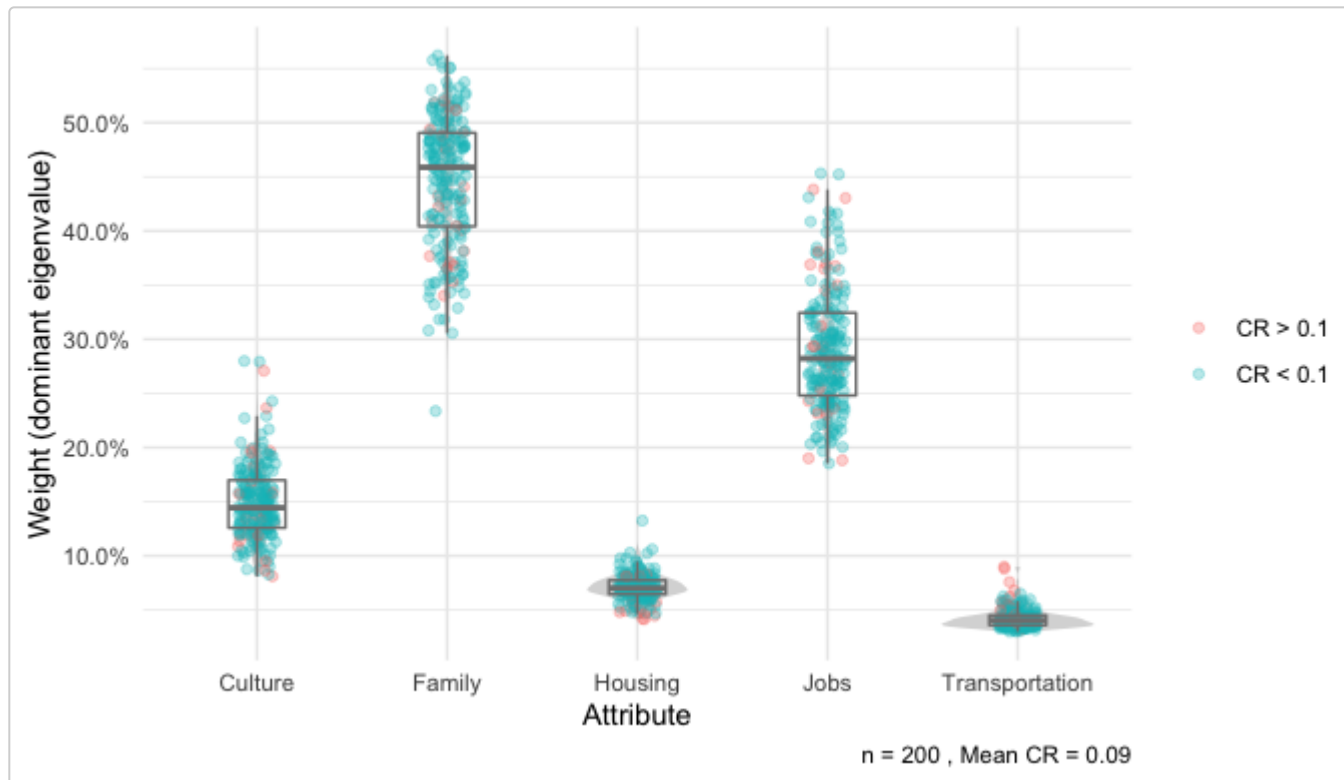
Consistency Ratios under different number of iterations with Harker's method

```
it <- 1
thres <- 0.1
cr.df1 <- data.frame(cr = city200 %>%
  ahp.mat(atts, negconvert = TRUE) %>%
  ahp.harker(atts, iterations = it, stopcr = 0.1, limit = T, round = T, printiter = F) %>%
  ahp.cr(atts))

cr.df2 <- cr.df1 %>%
  mutate(rowid = 1:nrow(city200), cr.dum = as.factor(ifelse(. <= thres, 1, 0))) %>%
  select(cr.dum, rowid)
```



```
city200 %>%
  ahp.mat(atts = atts, negconvert = TRUE) %>%
  ahp.harker(atts, iterations = it, stopcr = 0.1, limit = T, round = T, printiter = F) %>%
  ahp.indpref(atts, method = "eigen") %>%
  mutate(rowid = 1:nrow(city200)) %>%
  left_join(cr.df2, by = 'rowid') %>%
  gather(cult, fam, house, jobs, trans, key = "var", value = "pref") %>%
  ggplot(aes(x = var, y = pref)) +
  geom_violin(alpha = 0.6, width = 0.8, color = "transparent", fill = "gray") +
  geom_jitter(alpha = 0.3, height = 0, width = 0.1, aes(color = cr.dum)) +
  geom_boxplot(alpha = 0, width = 0.3, color = "#808080") +
  scale_x_discrete("Attribute", label = dict) +
  scale_y_continuous("Weight (dominant eigenvalue)",
                    labels = scales::percent, breaks = c(seq(0,0.7,0.1))) +
  guides(color=guide_legend(title=NULL))+
  scale_color_discrete(breaks = c(0,1),
                    labels = c(paste("CR >", thres),
                               paste("CR <", thres))) +
  labs(NULL, caption =paste("n =", nrow(city200), ", ", "Mean CR =", round(mean(cr),3)))+
  theme_minimal()
```



Individual preference weights with respect to goal (1 iteration)

Let's take a look at how applying Harker's method affects the overall aggregated priorities of the population.

```
options(scipen = 99)
inconsistent <- city200 %>%
  ahp.mat(atts = atts, negconvert = TRUE) %>%
  ahp.aggpref(atts, method = "eigen")

consistent <- city200 %>%
  ahp.mat(atts = atts, negconvert = TRUE) %>%
  ahp.harker(atts, iterations = 5, stopcr = 0.1, limit = T, round = T, printiter = F) %>%
  ahp.aggpref(atts, method = "eigen")

true <- t(ahp.indpref(sample_mat, atts, method = "eigen"))

aggpref.df <- data.frame(Attribute = atts, true, inconsistent, consistent) %>%
```

```
mutate(error.incon = abs(true - inconsistent),
       error.con = abs(true - consistent))
```

```
aggpref.df
```

```
## Attribute true inconsistent consistent error.incon error.con
## 1 cult 0.1522 0.1526 0.1427 0.000443 0.00945
## 2 fam 0.4335 0.4483 0.4388 0.014818 0.00537
## 3 house 0.0716 0.0705 0.0698 0.001031 0.00178
## 4 jobs 0.3050 0.2758 0.2886 0.029215 0.01638
## 5 trans 0.0378 0.0397 0.0424 0.001835 0.00454
```

Here I present the aggregated weights of the pairwise matrices without and with treatment with Harker's method, the aggregated priorities derived from the true weights of the sample, as well as the deviation of the priorities from the true weights. Because improving the consistency of the matrix does not necessarily increase the validity of the matrix, it is imperative that researchers consider other ways to improve consistency, ideally asking respondents to reconsider their choices, whenever inconsistency arises.

While there are strong arguments against replacing inconsistent values without the decision-maker's consent for the sake of satisfying the consistency ratio criterion of  $CR < 0.1$  (see Saaty and Tran (2007)), it is often not possible for survey executers to resolicit answers from their respondents after AHP analysis, whereas truncating inconsistent decisions may make the dataset unrepresentative of the population. Researchers should think carefully and explain fully the methods used to process AHP data.

## Imputing missing pairwise comparison matrices

Missing data is a common feature in surveys. Harker's method was originally developed to complete incomplete pairwise comparison matrices, and can be implemented here using the same strategy as `ahp.harker`.

```
missing.df <- city200[1:10,]
for (i in 1:10){
  missing.df[i, round(runif(1,1,10))] <- NA
  if (i > 7){
    missing.df[i, round(runif(1,2,10))] <- NA
  }
}
missing.df[,1:7]
```

```
##      cult_fam cult_house cult_jobs cult_trans fam_house fam_jobs fam_trans
## 1         2         -2         2         -6         -4         -4         -8
## 2         2         -4         1         NA         -4         -2         -8
## 3         4         -2         1         -3         NA         -3         -5
## 4         8         -4         3         NA         -8         1         -7
## 5         3         -3         5         -6         -8         1         -4
## 6         6         -4         2         -4         -7         -2         NA
## 7         7         -5         -3         -3         -8         1         NA
## 8         5         -4         NA         -5         -6         NA         -8
## 9         3         NA         2         -4         -6         NA         -6
## 10        7         -3         NA         NA         -8         -3         -5
```

To demonstrate the imputation function, I have randomly made some of the weights missing in a dataframe with ten observations. For rows 7 - 10, two weights are missing, and one are missing for others.

```
atts <- c("cult", "fam", "house", "jobs", "trans")
imputed <- missing.df %>%
  ahp.mat(atts, negconvert = TRUE) %>%
  ahp.missing(atts, round = T, limit = T)

actual <- city200 %>%
  ahp.mat(atts, negconvert = TRUE)

list(actual[[5]], imputed[[5]])
```

```
## [[1]]
##      cult  fam house jobs trans
## cult  1.000 0.333 3.000 0.200   6
## fam   3.000 1.000 8.000 1.000   4
## house 0.333 0.125 1.000 0.250   3
## jobs  5.000 1.000 4.000 1.000   6
## trans 0.167 0.250 0.333 0.167   1
##
## [[2]]
##      cult  fam house jobs trans
## cult  1.000 0.333 3.000 0.200   6
## fam   3.000 1.000 8.000 1.000   4
## house 0.333 0.125 1.000 0.250   3
```

```
## jobs 5.000 1.000 4.000 1.000 9
## trans 0.167 0.250 0.333 0.111 1
```

```
list(ahp.cr(actual, atts)[[5]], ahp.cr(imputed, atts)[[5]])
```

```
## [[1]]
## [1] 0.106
##
## [[2]]
## [1] 0.0966
```

```
list(actual[[8]], imputed[[8]])
```

```
## [[1]]
##      cult  fam house jobs trans
## cult  1.00 0.200  4.0 0.333   5
## fam   5.00 1.000  6.0 3.000   8
## house 0.25 0.167  1.0 0.250   2
## jobs  3.00 0.333  4.0 1.000   7
## trans 0.20 0.125  0.5 0.143   1
##
## [[2]]
##      cult  fam house jobs trans
## cult  1.00 0.200  4.0 0.333   5
## fam   5.00 1.000  6.0 1.000   8
## house 0.25 0.167  1.0 0.250   2
## jobs  3.00 1.000  4.0 1.000   7
## trans 0.20 0.125  0.5 0.143   1
```

```
list(ahp.cr(actual, atts)[[8]], ahp.cr(imputed, atts)[[8]])
```

```
## [[1]]
## [1] 0.0617
##
```

```
## [[2]]
## [1] 0.0498
```

Here, similar to `ahp.harker`, `ahp.missing` replaces the NA values in the pairwise comparison matrices with the most consistent pairwise choice available, thus the consistency ratio is increased after imputing missing values. The randomness and inconsistency of each decision-maker cannot be accounted for in the algorithm. Missing value imputation should be avoided whenever possible, but is a valid alternative if missing values are reasonably small in number.

## AHP on steroids: a canned routine

As of version 0.3.0, you can quickly execute most of the functions described in this vignette with a canned routine. Here I demonstrate how the `city200` dataset is processed using the AHP to output the aggregated and individual priorities of individuals, using the simplest arithmetic mean method.

```
canned <- ahp(df = city200,
             atts = c('cult', 'fam', 'house', 'jobs', 'trans'),
             negconvert = TRUE,
             reciprocal = TRUE,
             method = 'arithmetic',
             aggmethod = "arithmetic",
             qt = 0.2,
             censorcr = 0.1,
             agg = TRUE)

## [1] "Number of observations censored = 70"

head(canned$indpref)

##   cult  fam  house  jobs  trans  CR   top1   top2   top3
## 1 0.181 0.433 0.0904 0.260 0.0355 0.0614  fam_jobs  fam_trans  cult_jobs
## 2 0.227 0.394 0.0846 0.253 0.0418 0.0297  cult_house  fam_trans  cult_jobs
## 3 0.153 0.485 0.0860 0.223 0.0524 0.0634  fam_trans  cult_trans  fam_jobs
## 4 0.131 0.436 0.0695 0.327 0.0364 0.0932  cult_fam  cult_jobs  cult_house
## 5 0.152 0.489 0.0644 0.255 0.0391 0.0617  cult_jobs  cult_house  cult_fam
## 6 0.164 0.423 0.0785 0.293 0.0421 0.0383  fam_trans  cult_house  cult_jobs
```

You can also see clearly the individual priorities calculated using the arithmetic mean method, the consistency ratio of each individual, and which three pairwise comparisons are the most inconsistent.

```
canned$aggpref
```

```
##      AggPref SD.AggPref
## cult  0.1581  0.02970
## fam   0.4430  0.05422
## house 0.0757  0.00768
## jobs  0.2820  0.04593
## trans 0.0412  0.00559
```

You can also quickly censor inconsistent observations and estimate the aggregated preference weight of the population, and its standard deviation. The unique command here in this canned routine is `sensorcr`, where you may specify the consistency ratio censoring threshold, which allows you to remove observations a higher consistency ratio than you specified. Usually, researchers censor observations with a CR higher than 0.1. But sometimes it is different depending on your discipline. Note that specifying `qt` only affects `aggpref` calculation, and does not censor the observations in `$indpref`.

For many researchers, it would be useful if an ID identifier was included in the input dataframe, so that one can directly join the output to the existing dataset for further analysis. For example, here in this case the data in `city200` was coded with the name of an individual and their educational attainment. By specifying `ID`, I can retain a column or many columns containing an individuals' characteristics, and extract their preferences by educational attainment.

To add a bit of flavour to this example, I am going to use a package called `randomNames` that can be installed from CRAN by Betebenner (2017).

```
library(randomNames)

ed1 <- c("No High School", "High School", "Undergraduate", "Postgraduate")

edunames <- tibble(edu = factor(rep(ed1,50)),
                  names = randomNames(200, which.names = "first"),
                  catowner = c(rep(TRUE,100), rep(FALSE,100)))

citynames <- cbind(edunames, city200)
head(citynames)
```

```

##           edu  names catowner cult_fam cult_house cult_jobs cult_trans
## 1 No High School Ashley      TRUE      2      -2      2      -6
## 2   High School   Nova      TRUE      2      -4      1      -4
## 3 Undergraduate  Silas      TRUE      4      -2      1      -3
## 4 Postgraduate  Thomas      TRUE      8      -4      3      -4
## 5 No High School Joseph      TRUE      3      -3      5      -6
## 6   High School Aundrea      TRUE      6      -4      2      -4
##  fam_house fam_jobs fam_trans house_jobs house_trans jobs_trans
## 1      -4      -4      -8      4      -3      -8
## 2      -4      -2      -8      4      -3      -7
## 3      -7      -3      -5      4      -3      -6
## 4      -8      1      -7      4      -3      -9
## 5      -8      1      -4      4      -3      -6
## 6      -7      -2      -4      4      -3      -6

```

Here we have created a fictitious sample of the population with their pairwise comparisons of which city to live in. We can specify which columns to keep. We can also specify how the pairwise comparisons were ordered directly to the AHP routine so that it saves you a step from extracting the columns you need in the order you want.

```

named <- ahp(df = citynames,
             atts = c('cult', 'fam', 'house', 'jobs', 'trans'),
             negconvert = TRUE,
             reciprocal = TRUE,
             method = 'arithmetic',
             aggmethod = "arithmetic",
             qt = 0.2,
             censorcr = 0.1,
             agg = FALSE,
             ID = c("edu", "names")
             )

```

```

## Error in ahp(df = citynames, atts = c("cult", "fam", "house", "jobs", : Non numeric element(s)
in catowner column. Missing or miscoded data?

```

```

head(named)

```



```
## Error in head(named): object 'named' not found
```

Oops! There seems to be an irrelevant `catowner` column (that I've added) that you just want the routine to ignore. Fortunately, we could avoid this error by directly specifying the names and order of the columns that contain the pairwise comparisons using the `col` argument. The `col` argument also reorders your `data.frame` so that you don't have to do it manually if you want.

```
columns <- c("cult_fam", "cult_house", "cult_jobs", "cult_trans",
            "fam_house", "fam_jobs", "fam_trans",
            "house_jobs", "house_trans",
            "jobs_trans")
```

```
named <- ahp(df = citynames,
            atts = c('cult', 'fam', 'house', 'jobs', 'trans'),
            negconvert = TRUE,
            reciprocal = TRUE,
            method = 'arithmetic',
            aggmethod = "arithmetic",
            qt = 0.2,
            censorcr = 0.1,
            agg = FALSE,
            ID = c("edu", "names"),
            col = columns
            )
```

```
## [1] "Number of observations censored = 70"
```

```
head(named)
```

```
##           edu names cult  fam house jobs trans  CR      top1
## 1 No High School Ashley 0.181 0.433 0.0904 0.260 0.0355 0.0614  fam_jobs
## 2   High School   Nova 0.227 0.394 0.0846 0.253 0.0418 0.0297  cult_house
## 3 Undergraduate  Silas 0.153 0.485 0.0860 0.223 0.0524 0.0634  fam_trans
## 4 Postgraduate  Thomas 0.131 0.436 0.0695 0.327 0.0364 0.0932  cult_fam
## 5 Postgraduate  Nydia 0.152 0.489 0.0644 0.255 0.0391 0.0617  cult_jobs
## 6 No High School  Kyle 0.164 0.423 0.0785 0.293 0.0421 0.0383  fam_trans
##           top2      top3
```

```
## 1 fam_trans  cult_jobs
## 2 fam_trans  cult_jobs
## 3 cult_trans  fam_jobs
## 4 cult_jobs  cult_house
## 5 cult_house  cult_fam
## 6 cult_house  house_jobs
```

That way, it would only be fitting to show you how to summarize the mean of the priority weights of `cult` across groups.

```
named %>%
  group_by(edu) %>%
  dplyr::summarize(Mean = mean(cult, na.rm=TRUE))

## # A tibble: 4 x 2
##   edu          Mean
##   <fct>        <dbl>
## 1 High School  0.163
## 2 No High School 0.158
## 3 Postgraduate 0.153
## 4 Undergraduate 0.160
```

## Appendix I: data generation process

The dataset `city200` is generated based on Saaty's example of choosing a city to live in described in Saaty (2004). I recreated Saaty's example of criteria weights by simulating 200 decision-makers who make their choices based on the underlying true weights. The dataset is generated using a normal random sample from the Saaty scale with the mean set as the true weight and the standard deviation set manually. With a higher standard deviation, it is expected that the pairwise comparison will be less consistent. The methods employed later will reveal which pairwise comparison is less consistent than others.

```
## Defining attributes
set.seed(42)
atts <- c("cult", "fam", "house", "jobs", "trans")

colnames <- c("cult_fam", "cult_house", "cult_jobs", "cult_trans",
             "fam_house", "fam_jobs", "fam_trans",
```

```

    "house_jobs", "house_trans",
    "jobs_trans")

## True weights derived from Saaty's example
weight <- c(5,-3,2,-5,
           -7,-1,-7,
           4,-3,
           -7)

## Defining the saaty scale
saatyscale <- c(-9:-2, 1:9)
nobs <- 200

## saatyprob creates a list of probabilities in the saaty scale for being sampled given
## the position of the weight in the weight list (x) and standard deviation (sd)

saatyprob <- function(x, sd) dnorm(saatyscale, mean = weight[x], sd = sd)

## Standard deviation set on saatyprob(x, *sd*)
cult_fam <- sample(saatyscale, nobs, prob = saatyprob(1, 2), replace = TRUE)
cult_house <- sample(saatyscale, nobs, prob = saatyprob(2, 1), replace = TRUE)
cult_jobs <- sample(saatyscale, nobs, prob = saatyprob(3, 2), replace = TRUE)
cult_trans <- sample(saatyscale, nobs, prob = saatyprob(4, 1.5), replace = TRUE)
fam_house <- sample(saatyscale, nobs, prob = saatyprob(5, 2), replace = TRUE)
fam_jobs <- sample(saatyscale, nobs, prob = saatyprob(6, 1.5), replace = TRUE)
fam_trans <- sample(saatyscale, nobs, prob = saatyprob(7, 2.5), replace = TRUE)
house_jobs <- sample(saatyscale, nobs, prob = saatyprob(8, 0.5), replace = TRUE)
house_trans <- sample(saatyscale, nobs, prob = saatyprob(9, 0.5), replace = TRUE)
jobs_trans <- sample(saatyscale, nobs, prob = saatyprob(10, 1), replace = TRUE)

city200 <- data.frame(cult_fam, cult_house, cult_jobs, cult_trans,
                     fam_house, fam_jobs, fam_trans,
                     house_jobs, house_trans,
                     jobs_trans)

head(city200[,1:7])

##   cult_fam cult_house cult_jobs cult_trans fam_house fam_jobs fam_trans
## 1         2         -2         2         -6         -4         -4         -8
## 2         2         -4         1         -4         -4         -2         -8

```

## 3	4	-2	1	-3	-7	-3	-5
## 4	8	-4	3	-4	-8	1	-7
## 5	3	-3	5	-6	-8	1	-4
## 6	6	-4	2	-4	-7	-2	-4

Here, we have simulated the responses of 200 individual decision-makers regarding the attributes they consider when deciding which city to live in, based on the true weights from Saaty's journal article, with some added random deviations from the weight.

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